Syllabus

Course: Discovery-Based Mathematics, Part II
Presenter: Paul Lawrence

Required: Discovery-Based Math Manipulatives Kit ($69 materials fee) and Discovery-Based Mathematics: Teaching Elementary Mathematics, Part I

Course Overview

These numbers don’t lie. Test scores soar when students have a true understanding of number sense. Discovery-Based Mathematics, Part 2 is the second hands-on, inquiry-based approach to math presented by Paul Lawrence. In this course, Lawrence focuses on fractions and decimals. Through discovery-based experiences, students develop true conceptual understanding, rather than learning procedures without a basis in understanding.

Paul Lawrence leads educators through easy-to-implement well-sequenced activities that build foundational and conceptual understanding of fractions and decimals. Using a variety of manipulatives, Lawrence demonstrates the importance of hands-on discovery-based learning to move students from concrete to iconic to symbolic representations, before introducing procedures. His methods address differentiated instruction through kinesthetic and visual as well as abstract learning styles and by presenting multiple strategies for concepts. Educators follow along, using the materials in the Discovery-Based Math Manipulatives Kit, as they do the same activities workshop participants do. Educators learn techniques, activities, and games to assess students’ skills and concept understanding so that lessons can be adjusted to meet the needs of all learners.

Required Discovery-Based Math Manipulatives Kit ($69)

The supplementary kit includes a custom-tailored handbook to follow the online courses, as well as correlating handouts, Discovery Templates, a Self-Study Guide, and the patented “Communicator,” along with many manipulatives for hands-on activities. (The kit materials are packaged in a convenient carrying case.)

Course Objectives

After completing this course, educators will know:

- Reasons why discovery-based strategies are essential components in all math classrooms
How to use base ten blocks, connecting cubes, and grid templates to teach conceptual understanding and algorithm understanding of addition, subtraction, multiplication, and division of fractions and decimals

Techniques to transfer concrete understanding to mastery of traditional algorithms

How to use response devices, such as the Communicator™, to monitor understanding, adjust lessons, and formulate questions that reflect students’ current level of understanding and meet differentiated learning needs

How to incorporate techniques that build foundational understanding, number sense, and estimation skills so that students can effectively know when to use mental math, paper and pencil, or estimation and a calculator

Understanding of the appropriate uses of a calculator

Techniques for developing critical higher-order thinking skills and writing within the math curriculum

How to use inquiry-based techniques to foster student understanding, self-esteem, and self-confidence

Why this discovery-based approach to math will have a profound impact on student test scores

Student Learning Outcomes

After completing this course, educators will apply the following skills:

- Implement constructivist strategies
- Use manipulatives to teach conceptual understanding, fact mastery, and algorithmic understanding
- Segue students through concrete, iconic, and symbolic stages of learning
- Use response devices to monitor understanding, affect modifications in lessons, and formulate questions for class discussion
- Build students’ foundational understanding, number sense, and estimation skills
- Employ inquiry-based techniques that foster student understanding, self-esteem, and self-confidence

Unit 1: Concepts of Fractions and Decimals: Part 1

This is the first of three units on fractions and decimals. Paul Lawrence begins with an overview of what the units are leading up to by modeling how to guide students to choose which fraction problems are best done with mental math, paper and pencil, estimation, and calculators. As with other topics, Lawrence offers teachers several techniques to help students understand fractions, compare fractions, and estimate, that use both area and linear models. Lawrence then models making unit sticks to help students see fraction equivalences. He also
demonstrates how to use the unit sticks to give students practice with estimating using nonstandard measurement.

Unit Objectives

After completing this unit, educators will know:

- Hands-on methods for teaching fractions and fraction equivalences
- How to teach fractions using linear and area models
- How to create unit sticks using construction paper and tape
- Ways to use the unit stick to explore fraction equivalences and estimation
- Classroom management techniques that support honest answers for estimating activities

Student Learning Outcomes

After completing this unit, educators will apply the following skills:

- Use hands-on methods for teaching fractions and fraction equivalences
- Teach fractions using linear and area models
- Create fraction unit sticks with students using construction paper and tape
- Apply the unit stick to teach fraction equivalences and estimation
- Use classroom management techniques to support estimating activities

Unit 2: Concepts of Fractions and Decimals: Part 2

In the second of three units on fractions and decimals, Paul Lawrence offers teachers several techniques that they can use to help their students understand simplifying fractions and write equivalence and inequality statements. He models using a unit stick in conjunction with a giant inch template that will help students move from area to linear models when comparing fractions. He further demonstrates how teachers can use simulated rulers to compare fractions and how the fraction number template can be used to help students compare fifths, thirds, sevenths, and hundreds as well as halves, fourths, eights, and sixteenths. Finally, he demonstrates how to use a calculator to simplify fractions after students have understood the concept of simplification.

Unit Objectives

After completing this unit, educators will know:

- Fractional relationships using rulers
- How to use rulers to illustrate fractional relationships and at the same time teach students to read a ruler
- How to use the Fraction Stick Template
- How to teach students to simplify fractions
- When to use a calculator to simplify fractions

**Student Learning Outcomes**

After completing this unit, educators will apply the following skills:

- Teach fractional relationships using rulers
- Use rulers to illustrate fractional relationships and at the same time teach students to read a ruler
- Teach fraction equivalents using the Fraction Stick Template
- Teach students to simplify fractions algebraically
- Use calculators appropriately to simplify fractions

**Unit 3: Concepts of Fractions and Decimals: Part 3**

In the third unit on fractions and decimals, Paul Lawrence explores fractions and decimals using concrete and iconic representations for single units (divided into fractional parts) and groups of things. He models how to move from fractional equivalences to decimals, including how to use a calculator to demonstrate the traditional algorithm (divide the numerator by the denominator). Through a variety of templates with line segments, shapes, and grids, he shows teachers how to help students visualize decimals and discover how to compare them. Finally he introduces methods for teaching fractions as groups of things.

**Unit Objectives**

After completing this unit, educators will know:

- How to use the Fraction Stick template to compare fractions
- How to use the Fraction Stick template to teach fraction-to-decimal conversions
- When and how to use a calculator to do the fraction-to-decimal procedure
- How to use linear, area, and money to model tenths
- How to use grids to develop concepts of tenths, hundredths, and thousandths
- Templates to use to teach fractions as groups of things

**Student Learning Outcomes**

After completing this unit, educators will apply the following skills:
• Compare fractions using the Fraction Stick template
• Convert fractions to decimals using the Fraction Stick template
• Use a calculator to do the fraction-to-decimal procedure
• Model tenths using linear, area, and money
• Develop concepts of tenths, hundredths, and thousandths
• Teach fractions as groups of things using 24 and 60 penny templates

Unit 4: Addition and Subtraction of Fractions with Same and Compatible Denominators

In this unit, Paul Lawrence focuses on techniques to teach adding and subtracting fractions with the same and compatible denominators. He demonstrates how concepts can be presented using Fraction Tiles, rulers, and Fraction Stick equivalence charts. He connects the techniques he introduces to more traditional approaches to instruction that use algorithms and he offers further guidance on how to help students learn how to solve problems using mental math. Finally, he introduces several games to reinforce the concepts covered in this unit.

Unit Objectives

After completing this unit, educators will know:

• How to use Fraction Tiles, rulers, and equivalence charts to teach addition and subtraction of fractions
• How to transfer concepts from iconic experiences to mental math to solve addition and subtraction problems involving fractions
• Games to enhance and practice addition and subtraction of fractions with like and compatible denominators

Student Learning Outcomes

After completing this unit, educators will apply the following skills:

• Use a variety of manipulatives and templates to teach addition and subtraction of fractions with like and compatible denominators
• Transfer concepts from iconic experiences to mental math to solve addition and subtraction problems involving fractions
• Use games to enhance and practice addition and subtraction of fractions with like and compatible denominators
Unit 5: Addition and Subtraction of Fractions with Non-Compatible and Overlapping Denominators

In this unit, Paul Lawrence focuses on adding and subtracting fractions with non-compatible and overlapping denominators. He develops conceptual understanding through multiple strategies using Fraction Files and Fraction Sticks and by counting on. He demonstrates the importance of using number sense to estimate and when and how calculators should be used with fractions in the classroom. The unit concludes with another “You Decide” page of problems and guidelines to determine whether problems should be solved using mental math, paper and pencil, or estimation and a calculator.

Unit Objectives

After completing this unit, educators will know:

- How to use Fraction Tiles and Fraction Sticks to teach addition and subtraction of fractions with non-compatible and overlapping denominators
- When and how to use calculators appropriately to solve fraction problems
- How to teach subtraction of fractions by counting on
- How to use number sense to estimate with fractions
- Techniques for determining the best method to solve fraction problems

Student Learning Outcomes

After completing this unit, educators will apply the following skills:

- Teach addition and subtraction of fractions with non-compatible and overlapping denominators using Fraction Tiles and Fraction Sticks
- Apply number sense to estimate addition and subtraction of fractions
- Teach the appropriate use of calculators to solve fraction problems
- Teach subtraction of fractions by counting on
- Teach strategies to determine whether mental math, paper and pencil, or a calculator should be used to solve problems based on the context of the problem

Unit 6: Multiplication of Fractions

Paul Lawrence focuses on techniques that can help students understand the concept of the multiplication of fractions. Connecting what students already know about multiplication of whole numbers to fractions, Lawrence models how to use arrays and Fraction Tiles to multiply fractions. He also demonstrates using iconic drawings to illustrate problems without using manipulatives. As in other units, Lawrence shows how and when calculators should be used with fractions in the classroom. This unit covers multiplication of proper fractions times proper
fractions, proper fractions times whole numbers and mixed numbers, and mixed numbers times mixed numbers.

**Unit Objectives**

After completing this unit, educators will know:

- How to use area models to understand multiplication of fractions
- How to use Fractions Tiles to understand multiplication of fractions
- How to use hand-drawn models to determine products of proper fractions
- How to guide students to discover the algorithm
- Why students should learn to estimate products of fractions
- When and how to use calculators to determine the product of two fractions

**Student Learning Outcomes**

After completing this unit, educators will apply the following skills:

- Teach multiplication of fractions using area models
- Teach multiplication of fractions using Fractions Tiles
- Use hand-drawn models to determine products of proper fractions
- Guide students to discover the algorithm for multiplying fractions
- Teach students to estimate products of fractions
- Teach when and how to use calculators for multiplying fractions

**Unit 7: Division of Fractions**

As with previous topics, Paul Lawrence begins this unit on division of fractions with concrete experiences. He models the concept using Fraction Tiles. Lawrence develops division of fractions from simple to more difficult (proper fractions, proper fractions into whole numbers and mixed numbers, and mixed numbers into mixed numbers) with lots of practice before introducing the procedure. He also models estimating answers as the first step to using calculators with division of fractions. Finally, he offers guidelines for applying number sense to division of fractions to determine whether problems should be solved using mental math, paper and pencil, or estimation and a calculator.

**Unit Objectives**

After completing this unit, educators will know:

- How to use Fraction Tiles to introduce and practice division of fractions
- How to develop the standard fraction division algorithm through discovery
• How to determine and apply efficient methods and strategies to find quotients by using mental math, paper and pencil, or estimation and a calculator based on the context of the problem
• How to apply number sense to estimate answers
• How to use a fraction capable calculator to solve fraction problems after estimating

**Student Learning Outcomes**

After completing this unit, educators will apply the following skills:

• Use Fraction Tiles to introduce and practice division of fractions
• Develop the standard fraction division algorithm through discovery
• Model how to apply efficient methods and strategies to find quotients by using mental math, paper and pencil, or estimation and a calculator based on the context of the problem
• Apply number sense to estimate answers
• Teach students to use a fraction capable calculator to solve fraction problems after estimating

**Unit 8: Addition and Subtraction of Decimals**

Paul Lawrence addresses addition and subtraction of decimals using the same sequence of instruction to build understanding from iconic to symbolic without ever saying, “just line up the decimals.” He models linear and area models to cement understanding of both addition and subtraction of decimals. This methodology builds a foundation for algebra because students are adding and subtracting like terms. Lawrence applies strategies from whole numbers, including partial sums, trade first, same change, estimation, and games to teach decimals. He demonstrates techniques and offers guidelines that can help students apply number sense to decide which method is most appropriate to use in solving individual problems.

**Unit Objectives**

After completing this unit, educators will know:

• How to use area models to visualize and add and subtract units and tenths, and units, tenths, and hundredths
• How to use linear models to visualize and add and subtract units and tenths, and units, tenths, and hundredths
• How to teach “Trade First Algorithm” to subtract units and tenths, and units, tenths, and hundredths
• How to apply estimation skills to adding decimals
• How to choose appropriate methods to subtract decimals
Student Learning Outcomes

After completing this unit, educators will apply the following skills:

- Use area models to visualize and add and subtract units and tenths, and units, tenths and hundredths
- Use linear models to visualize and add and subtract units and tenths, and units, tenths and hundredths
- Teach Trade First Algorithm to subtract units and tenths, and units, tenths and hundredths
- Apply estimation skills to adding decimals
- Teach students to choose appropriate methods to subtract decimals
- Introduce games using the four-step model
- Play the “Units, Tenths and Hundredths” Game

Unit 9: Multiplication and Division of Decimals: Part 1

Paul Lawrence approaches multiplication and division of decimals in the same way he introduced multiplication and division of whole numbers. He demonstrates linear and area models to increase student understanding, without introducing the algorithm. He also models techniques to apply number sense to estimating answers and explains why students should always be asked to estimate before using a calculator. Lawrence offers guidelines that can help students decide which methods are most appropriate to use in solving individual problems.

Unit Objectives

After completing this session, educators will know:

- How to teach multiplication and division of decimals using area and linear models
- How and why number sense and estimation should be used to determine a quotient
- When to use calculators with multiplication and division of decimals
- Practice formats that require students to use number sense, operational knowledge of decimals, and problem solving to determine solutions
- How to teach students to choose efficient methods and strategies to solve mixed sets of operations with whole numbers, fractions, and decimals by choosing from mental math, paper and pencil, or estimation and a calculator
**Student Learning Outcomes**

After completing this unit, educators will apply the following skills:

- Teach multiplication and division of decimals using area and linear models
- Model how to use number sense and estimation to determine a quotient
- Use calculators with multiplication and division of decimals
- Practice formats that require students to use number sense, operational knowledge of decimals, and problem solving to determine
- Teach students to choose efficient methods and strategies to solve mixed sets of operations with whole numbers, fractions, and decimals by choosing from mental math, paper and pencil, or estimation and a calculator

**Unit 10: Multiplication and Division of Decimals: Part 2**

In this final unit, Paul Lawrence addresses two topics: how games can be used to teach math concepts and how to use open-ended questions. He explains the use of open-ended questions and evaluating written answers as a way to assess concept understanding and to prepare students to succeed on constructivist standardized test questions. Lawrence also shares a nine-step program for creating student-constructed responses. Finally, Lawrence reviews some of the overarching ideas for the entire course and urges teachers to commit to using discovery-based math techniques in their classrooms.

**Unit Objectives**

After completing this unit, educators will know:

- Games that will reinforce operations with decimals
- How and why writing and open-ended questions are important in the math curriculum
- How to develop student-constructed responses to open-ended questions
- Key concepts for the course

**Student Learning Outcomes**

After completing this unit, educators will apply the following skills:

- Play “Decimal Roulette,” “Decimal Get a 100,” and “Dancing with Decimals” games to reinforce decimal math skills
- Use open-ended questions in the math curriculum
- Assess student understanding by having them evaluate answers written by their peers
- Reinforce key principles from the course
Presenter’s Bio

Paul Lawrence has been involved in mathematics education for over forty years, teaching high school, working as a math supervisor for grades K-12, and lecturing part-time (as a visiting instructor) at Rutgers University. A past president of the Association of Math Teachers of New Jersey (AMTNJ), Lawrence has sat on committees to assess New Jersey state tests in mathematics. He is a frequent presenter at local, state, and national conferences, and he has conducted hundreds of workshops throughout the United States. Lawrence was named by AMTNJ as the Max Sobel Outstanding Mathematics Educator for 2000.

Methods of Instruction:

- Videos (presentations consisting of lecture, interviews, and workshop footage)
- Course handbook: *Discovery Based Mathematics: Teaching Elementary Mathematics Course 1*
- Reflection questions (open-ended questions in each unit that ask educators to reflect on the course content, their own practice, and their intentions for their practice)
- Quizzes (selected-response quizzes to assess understanding of the video presentations)

All steps listed under each topic must be completed to receive credit for the course. No partial credit is given.

Plagiarism Policy

KDS recognizes plagiarism as a serious academic offense. Plagiarism is the dishonest passing off of someone else’s work as one’s own and includes failing to cite sources for others’ ideas, copying material from books or the Internet, and handing in work written by someone other than the participant. Plagiarism will result in a failing grade and may have additional consequences. For more information about plagiarism and guidelines for appropriate citation, consult plagiarism.org.

KDS Rubric for Pass/Fail Option: CEU

Passing Requirements:

- 70 points or more
- No “unsatisfactory” in either category
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<td>Reflection questions</td>
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<td>-Participant includes no content from the course in his or her responses</td>
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<td>-Participant does not address the questions posed</td>
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