Course Overview

These numbers don’t lie: test scores soar when students have a true understanding of number sense. Discovery-Based Mathematics is a hands-on, inquiry-based approach to math that grounds student knowledge firmly in number sense and then develops conceptual understanding, so that students can do double-digit computation … in their heads!

Presenter Paul Lawrence leads educators through easy-to-implement, well-sequenced activities that build foundational and conceptual understanding in real, whole, and negative numbers and addition and subtraction of whole numbers. Using a variety of manipulatives, Lawrence demonstrates the importance of using hands-on discovery-based learning to move students from concrete to iconic to symbolic representations, before introducing procedures. His methods address kinesthetic and visual as well as abstract learning styles. Educators follow along, using the materials in the Discovery-Based Math Manipulatives Kit, as they do the same activities workshop participants do. Educators learn techniques, activities, and games to assess students’ skills and concept understanding, so that lessons can be adjusted to meet the needs of all learners.

Required Discovery-Based Math Manipulatives Kit

The supplementary kit includes a custom-tailored handbook to follow the online courses, as well as correlating handouts, Discovery Templates, a Self-Study Guide, and the patented “Communicator,” along with many manipulatives for hands-on activities. (The kit materials are packaged in a convenient carrying case.)

Presenters’ Bios

Paul Lawrence has been involved in mathematics education for over forty years, teaching high school, working as a math supervisor for grades K-12, and lecturing part-time (as a visiting instructor) at Rutgers University. A past president of the Association of Math Teachers of New Jersey (AMTNJ), Lawrence has sat on committees to assess New Jersey state tests in mathematics. He is a frequent presenter at local, state, and national conferences, and he has conducted hundreds of workshops throughout the United States. Lawrence was named by AMTNJ as the Max Sobel Outstanding Mathematics Educator for 2000.

Objectives

After completing this course, educators will know:
- Reasons why discovery-based strategies are essential components in all math classrooms
- How to use base ten blocks, connecting cubes, and geoboards to teach conceptual understanding, fact mastery, and algorithm understanding of addition, subtraction, multiplication, and division of whole numbers
- Why all students need to experience concepts through concrete, iconic, and then symbolic stages of learning
- Techniques of how to transfer concrete understanding to fact and algorithmic mastery of traditional algorithms
- How to use response devices, such as the Communicator, to monitor understanding, adjust lessons, and formulate questions that reflect students’ current level of understanding
- How to incorporate techniques that build foundational understanding, number sense, and estimation skills so that students can effectively know when to use mental math, paper and pencil, or estimation, or a calculator
- Understanding of the appropriate uses of a calculator
- Techniques for developing critical higher-order thinking skills and writing within the math curriculum
- How to use inquiry based techniques to foster student understanding, self-esteem, and self-confidence
- Reasons why this discovery-based approach to math will have a profound impact on student test scores

Student Learning Outcomes

After completing this course, educators will apply the following skills:

- Implement constructivist strategies
- Use manipulatives to teach conceptual understanding, fact mastery, and algorithmic understanding
- Segue students through concrete, iconic, and symbolic stages of learning
- Use response devices to monitor understanding, affect modifications in lessons, and formulate questions for class discussion
- Build students’ foundational understanding, number sense, and estimation skills
- Employ inquiry-based techniques that foster student understanding, self-esteem, and self-confidence

Unit 1: Classifying, Ordering, and Exploring Real Numbers: Part 1

Paul Lawrence illustrates various ways that discovery-based instructional strategies can be effectively applied to math instruction at the elementary level. In this presentation, Mr. Lawrence considers the changes that have come about in math education in recent years: a move from rote, procedure based, textbook learning to more constructivist, discovery based teaching and learning. He suggests that math teachers need to adopt a more hands-on approach to instruction in which students are presented with procedures only after they have mastered the concept that underlies the procedure. In this presentation, he addresses the use of a variety of math manipulatives and demonstrates how these can be effectively applied to helping students understand the base ten number system.
Unit Objectives

After completing this unit, educators will know:

- Reasons for using math manipulatives
- Uses of the Communicator™
- Understand discovery-based methods to help students understand the base ten number system
- How to use discovery templates
- How to help students understand how to order and compare sets of whole numbers

Student Learning Outcomes

After completing this unit, educators will apply the following skills:

- Employ math manipulatives in instruction
- Use the Communicator™ and discovery templates
- Use discovery-based methods to help students understand the base ten number system
- Guide students to illustrate, compare, and order sets of whole numbers

Unit 2: Classifying, Ordering, and Exploring Real Numbers: Part 2

Paul Lawrence considers how discovery-based learning can guide students to master essential math concepts including: rounding, ordering, exponential notation, prime numbers, and composite numbers. Lawrence models using variety of math manipulatives, such as math cubes, geoboards, and arrays, as well as templates to help students visualize numbers and other math concepts and develop critical higher order thinking and problem solving skills.

Unit Objectives

After completing this unit, educators will know:

- Why math manipulatives are essential for teaching math concepts
- How discovery-based methods can substantially enrich math instruction
- How geoboards and arrays can help teach essential math concepts
- Why discovery-based approaches help students develop important higher order thinking skills, e.g., problem solving and pattern seeking

Student Learning Outcomes

After completing this unit, educators will apply the following skills:

- Use math manipulatives including geoboards and arrays to teach essential math concepts
- Use discovery-based methods to enrich math instruction and help students develop important
higher order thinking skills, e.g., problem solving and pattern seeking

### Unit 3: Classifying, Ordering, and Exploring Real Numbers: Part 3

In this unit, Paul Lawrence introduces the idea of a function machine. He explores ways function machines can be used for factorization, finding greatest common factors, and finding least common multiples. Lawrence also demonstrates an inquiry-based approach that leads students to discover divisibility rules.

**Unit Objectives**

After completing this unit, educators will know:

- What a function machine is and how it can be used to teach essential math concepts
- Strategies to help students discover and understand divisibility

**Student Learning Outcomes**

After completing this unit, educators will apply the following skills:

- Use function machines to teach essential math concepts
- Employ strategies to help students understand divisibility

### Unit 4: Exploring Negative Numbers, Scientific Notation, and Order of Operations

In this unit, Paul Lawrence explores how teachers can help students understand comparisons between negative and positive numbers through their familiarity with vertical number lines, (i.e., thermometers). He demonstrates how scientific notation can be taught and used in calculations without introducing the idea of moving the decimal point. Finally, he addresses order of operations and provides a model for leading students to discover the correct order without relying on the acronym PEMDAS (Please Excuse My Dear Aunt Sally). He emphasizes the point that calculators can be effective tools to help students understand and explore concepts when used during concept discovery problem solving.

**Unit Objectives**

After completing this unit educators will know:

- How to introduce and help students visualize negative numbers
- How to effectively teach scientific notation and its use in calculations without using the idea of moving the decimal point
- How to use investigation to teach order of operations
- When to use PEMDAS in teaching the order of operations
Student Learning Outcomes

After completing this unit, educators will apply the following skills:

- Use vertical timelines to introduce and compare positive and negative numbers
- Teach scientific notation through discovery without using the procedure of moving the decimal point
- Explore order of operations through problem-solving investigations
- Use acronyms to summarize conclusions drawn

Unit 5: Addition and Subtraction of Whole Numbers: Part 1

Paul Lawrence explores how teachers can help students understand basic addition and subtraction of whole numbers. Using connecting cubes, geoboards, hundred blocks and charts, Lawrence demonstrates how to extend fact practice to include algebraic thinking skills long before the standard introduction to the algorithms. Lawrence explains the value of teaching subtraction as counting on, as a means to also practice making change and calculating elapsed time.

Unit Objectives

After completing this unit, educators will know:

- Why manipulatives should be used for understanding of basic addition and subtraction facts
- The sequence of steps in developing understanding to the symbolic stage
- The value of teaching subtraction as counting on
- How to use flats, rods, and units to add and subtract
- Multiple strategies for solving math problems

Student Learning Outcomes

After completing this unit, educators will apply the following skills:

- Use a variety of manipulatives to ground basic addition and subtraction facts in concrete and iconic experiences
- Develop understanding of addition and subtraction through the symbolic stage
- Teach subtraction as counting on
- Teach addition and subtraction using multiple strategies

Unit 6: Addition and Subtraction of Whole Numbers: Part 2
In this second unit on addition and subtraction of whole numbers, Paul Lawrence introduces several games that can be used to help students master addition and subtraction with whole numbers. He demonstrates how games build understanding of concepts and provide much needed practice in ways that students enjoy. He also addresses the important number-sense skill of estimating and why teachers should teach multiple strategies to solve mixed sets of multi-digit addition and subtraction problems.

Unit Objectives

After completing this unit, educators will know:

- Why games are important teaching tools
- The games “Win a Flat,” “Lose a Flat,” and “Column Addition”
- The importance of teaching multiple strategies for solving math problems
- How to help students develop their capacity to estimate answers to math problems

Student Learning Outcomes

After completing this unit, educators will apply the following skills:

- Use the games “Win a Flat,” “Lose a Flat,” and “Column Addition” to teach addition and subtraction
- Teach multiple strategies for solving math problems
- Teach students to estimate answers to math problems

Unit 7: Mastering Multiplication and Division Facts: Part 1

In this first of a three units on mastering multiplication and division facts, Lawrence leads teachers through several exercises to ground student understanding in concrete experiences. He uses Elinor Pinczes’ *One Hundred Hungry Ants*, manipulatives, and student-made booklets to make the abstract personal. Lawrence demonstrates how to use arrays to reinforce concepts of rows and columns and the commutative property of multiplication. He urges teachers to ensure that students have a firm grasp of the concept of multiplication before introducing the times tables. In doing so, he carefully moves from the concrete to the iconic and then the symbolic. Lawrence also stresses the importance of using more than one approach to reach the same end — i.e., to facilitate students’ mastery of the concept.

Unit Objectives

After completing this unit, educators will know:

- Why they should use manipulatives to teach multiplication
- Why it is important to use multiple strategies when teaching math concepts
- How to move from the concrete to the iconic to the symbolic to teach math concepts
Student Learning Outcomes

After completing this unit, educators will apply the following skills:

- Use concrete manipulatives to teach multiplication
- Use Elinor Pinczes’ *One Hundred Hungry Ants* to develop math concepts
- Develop students’ understanding of rows and columns
- Teach multiple strategies for understanding math concepts
- Teach strategies that move from concrete to iconic to symbolic

Unit 8: Mastering Multiplication and Division Facts: Part 2

In the second of a three-part session on mastering multiplication and division facts, Paul Lawrence continues to model multiple strategies for teaching multiplication. Having demonstrated using arrays and groups of things in part 1, Lawrence models the strategy of repeated addition. Once again, he shows how to use a calculator, this time as function machine for repeated addition to explore concepts. Lawrence reviews how to have students make their own individual multiplication booklets. Incorporating higher-order thinking skills, Lawrence introduces using open-ended math questions and writing about math to further understanding as well as analytic and writing skills. After students fully understand the concepts, Lawrence offers effective strategies for memorizing the multiplication tables for numbers 1 through 10. He shows how of the 100 facts, there are just 15 “hard” facts to memorize. Finally he demonstrates how teachers can employ geoboards to simultaneously teach rectangles, areas, and multiplication facts.

Unit Objectives

After completing this unit, educators will know:

- How to use a calculator as a function machine
- Why open-ended questions and discussion in math instruction are important
- Ways to use geoboards to teach multiple concepts
- How to construct practice pages that support conceptual understanding

Student Learning Outcomes

After completing this unit, educators will apply the following skills:

- Program and use a calculator as a function machine for repeated addition
- Use open-ended math questions, written answers, and analyzing the answers of others to promote concept comprehension, writing, and thinking skills
- Use geoboards to review or teach multiple concepts
- Develop practice pages that support concept understanding with iconic representations of algorithms
Unit 9: Mastering Multiplication and Division Facts: Part 3

In this third of three units on mastering multiplication and division facts, Paul Lawrence continues to model how teachers can help students master the concept of multiplication and extends his discussion to include division. He introduces several games and strategies to use with multi-digit multiplication. Lawrence shows how students can develop the capacity to use mental math to solve double-digit multiplication problems.

Unit Objectives

After completing this unit, educators will know:

- Games that reinforce instruction in multiplication and division
- Function machines that teach multiplication
- How to teach multiplication using partial product algorithm
- Steps to teach students to do mental math with multi-digit multiplication problems

Student Learning Outcomes

After completing this unit, educators will apply the following skills:

- Use games to reinforce instruction in multiplication and division
- Use function machines to teach multiplication
- Use partial product algorithm
- Implement a sequence of steps to teach students mental math with multi-digit multiplication problems

Unit 10: Mastering Multiplication and Division Beyond Facts: Part 1

This is the first of a two more units on multiplication and division, moving beyond basic facts to understanding multi-digit computation. Paul Lawrence demonstrates how to use arrays and partial product methods to do multi-digit multiplication problems. He also introduces ideas and suggestions for helping students learn to decide when a problem is best solved using pencil and paper, estimation, or a calculator.

Unit Objectives

After completing this unit, educators will know:

- How to use arrays to teach multi-digit multiplication
- How to use the partial product algorithm to teach multi-digit multiplication
- When and how to estimate and validate estimations for multi-digit multiplication problems
When and why students should use calculators

Student Learning Outcomes

After completing this unit, educators will apply the following skills:

- Use arrays to teach multi-digit multiplication
- Teach the partial product algorithm to compute multi-digit multiplication
- Employ strategies to estimate and validate estimations for multi-digit multiplication problems
- Have students use calculators appropriately

Unit 10: Mastering Multiplication and Division Beyond Facts: Part 2

This is the second of two units that move beyond basic multiplication and division. Paul demonstrates how teachers can use tile templates, play money and partial quotient and fare share methods to help their student understand and solve multi-digit division problems. Lawrence shares worksheets from his programs in which students need to apply number sense to decide the best method for solving problems along with computation practice using different strategies. Part 1 of the course concludes with Three Digit Fun, a game for students to practice multiple skills and strategies as they figure out how to make three digits into problems that equal the given answers.

Unit Objectives

After completing this unit, educators will know:

- How to use templates and acting out to teach multi-digit division
- Why multiple strategies should be used to teach multi-digit division
- How to apply estimating skills to multi-digit division problem practice
- When to have students use calculators
- How to use games to practice multiple math skills

Student Learning Outcomes

After completing this unit, educators will apply the following skills:

- Use templates and acting out to teach multi-digit division
- Teach fair share and partial quotient algorithm to division problems
- Apply estimating skills to multi-digit division problem practice
- Appropriately use calculators with division
- Use games to practice multiple math skills
Methods of Instruction

- Videos (presentations consisting of lecture, interviews, and classroom footage)
- Readings
- Reflection questions (open-ended questions at intervals throughout the video presentations where participants are asked to reflect on the course content, their own practice, and their intentions for their practice)
- Quizzes (selected-response quizzes to assess understanding of the video presentations)
- Optional discussion forum (a place to meet with colleagues to discuss the ideas presented in the course)
- Final (a sustained reflection on an overarching final question)

Plagiarism Policy

KDS recognizes plagiarism as a serious academic offense. Plagiarism is the passing off of someone else’s work as one’s own and includes failing to cite sources for others’ ideas, copying material from books or the Internet (including lesson plans and rubrics), and handing in work written by someone other than the participant. Plagiarism will result in a failing grade and may have additional consequences. For more information about plagiarism and guidelines for appropriate citation, consult plagiarism.org.

Percentage of Course Credit

- Reflection questions 30%
- Quizzes 10%
- Final 60%

In order to complete the requirements of the course, the participant must complete all course work (e.g., reflections, quizzes, and any midterm and/or final), including watching all videos and participating in all discussion forums. We do not award partial credit.

Grading Policy

A: 3.4 – 4.0
B: 2.7 – 3.3
F: >2.6
### Reflection/Quiz Rubric

<table>
<thead>
<tr>
<th>Activity</th>
<th>Distinguished (4)</th>
<th>Proficient (3)</th>
<th>Basic (2)</th>
<th>Unsatisfactory (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quizzes</td>
<td>90-100%</td>
<td>80-89%</td>
<td>70-79%</td>
<td>69% or below</td>
</tr>
<tr>
<td>Reflection Question</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Participant has provided rich detail and supporting examples from the course content.</td>
<td>Participant has included appropriate content from the course content.</td>
<td>Participant has included little that indicates consideration and comprehension of course content.</td>
<td>Participant has included little to no content indicating consideration and comprehension of course content.</td>
</tr>
<tr>
<td></td>
<td>Participant has made responses to prompts personally meaningful and relevant to his or her teaching practice.</td>
<td>Participant has made thoughtful comments in direct response to the prompts.</td>
<td>Participant has answered most questions directly but some too briefly.</td>
<td>Participant has not addressed the specific questions posed.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Final Reflection

Paul Lawrence urges teachers to spend much more time with manipulatives to cement foundational concepts before moving to icons, then attaching those icons to symbols. In 3-5 paragraphs, describe how you will integrate more use of manipulatives into your math instruction to achieve those specific ends. Clarify the connection between the specific manipulatives you have selected and the concepts they are meant to compel comprehension of.

### Final Rubric

<table>
<thead>
<tr>
<th>Step</th>
<th>Distinguished (4)</th>
<th>Proficient (3)</th>
<th>Basic (2)</th>
<th>Unsatisfactory (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Requirements of the Assignment</td>
<td>Participant has fulfilled all of the requirements of the assignment with marked creativity,</td>
<td>Participant has sufficiently fulfilled the requirements of the assignment.</td>
<td>Many requirements are met, but a few components are missing, while others are</td>
<td>The assignment is substantially incomplete.</td>
</tr>
<tr>
<td><strong>Content</strong></td>
<td>Supporting points are illustrated with examples and/or quotes.</td>
<td>Thesis and supporting ideas are usually rationalized by evidence, examples, or quotes.</td>
<td>Thesis and supporting ideas are supported by minimal evidence, examples, or quotes.</td>
<td>No supporting evidence for theses or subordinate ideas.</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Lesson plan/essay shows evidence of a deep understanding of course content and participant uses that understanding to create opportunities for students to authentically show what they have learned with appropriate assessments.</td>
<td>The lesson plan/essay and objectives, activities, and/or assessment show evidence of understanding of the course content.</td>
<td>The lesson plan/essay shows inadequate evidence that the learner understands the course content.</td>
<td>Little evidence in the lesson plan/essay or in objectives, activities, or assessments that the learner comprehends the course content.</td>
<td></td>
</tr>
<tr>
<td>Participant utilizes adequate course material.</td>
<td>Participant incorporates abundant course material.</td>
<td>Objectives, activities and/or assessments do not directly apply to the course content.</td>
<td>Objectives activities and/or assessments do not directly apply to the course content.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Quality of Writing</strong></th>
<th>No grammatical errors or typos.</th>
<th>Few grammatical errors or typos.</th>
<th>Distracting grammatical errors or typos.</th>
<th>Plentiful grammatical errors or typos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proper citation of sources.</td>
<td>Appropriate citation of sources.</td>
<td>Inconsistent or missing documentation of sources.</td>
<td>Missing documentation of sources.</td>
<td></td>
</tr>
<tr>
<td>Essay is organized around a sophisticated thesis or main idea.</td>
<td>Essay is organized around a thesis or main idea.</td>
<td>The main idea is not clear.</td>
<td>No main idea and/or main idea is irrelevant to the assignment.</td>
<td></td>
</tr>
<tr>
<td>Paragraphs are clearly organized around ideas relevant to the main idea and fully developed.</td>
<td>Paragraphs are organized around ideas relevant to the main idea.</td>
<td>Relevance to main idea of supporting paragraphs is not always clear.</td>
<td>No apparent paragraph organization.</td>
<td></td>
</tr>
</tbody>
</table>